Table 1. Area average Nusselt numbers for open area unit cell and $Re_D = 17\,000$

H/D	Center jet	Side jet	Corner jet	Entire array
6.0	62.3	62.7	63.0	62.8
1.0	83.8	80.6	76.9	79.2
0.25	84.6	78.3	74.2	77.1

shape than circular. However, the differences between the contours for the center and perimeter jets were not large (about 15%). The small differences resulted in small variations of less than 12% between the average Nusselt numbers for the center and perimeter jets. The expected trends in the average Nusselt number between the jets were observed, but with small variations. Therefore, perimeter jets do differ from center jets, but for the conditions studied the differences are small.

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Int. J. Heat Mass Transfer. Vol. 37, No. 18, pp. 3030–3033, 1994

0017-9310/94 $7.00+0.00

An investigation of a wave of temperature difference between solid and fluid phases in a porous packed bed

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(*Received* 16 *December* 1993)

INTRODUCTION

Non-thermal equilibrium flow of a fluid through a porous bed is a subject of permanent interest for analytical and numerical investigations. Most of analytical studies of the phenomenon were concentrated on the Schumann model of a packed bed, obtained in ref. [1]. The model ignores the conduction terms in the solid and gas (liquid) phase energy equations. Originally the thermal capacity term in the fluid phase energy equation was also neglected, but in some further studies the effect of the thermal capacity of the fluid was included in the analysis. Analytical solutions for the model for various input conditions have been obtained in refs. [2–5]. Analysis and comparison of analytical solutions for the two-phase model (two energy equations) and the single-phase model (local thermal equilibrium assumption, and, as a result, one energy equation) are presented in ref. [6]. In refs. [7–9] a very general set of volume-averaged governing equations for non-thermal equilibrium condensing forced flow through a latent heat storage porous bed was presented and comprehensive numerical investigations of the phenomenon were carried out.

Distinguished from the previous analytical investigations the present analysis is based on solution by the perturbation technique of the full energy equations for fluid and solid phases, without neglecting any terms in the equations.

STATEMENT OF THE PROBLEM

Assumptions made in the analysis are outlined in the following:

(1) heat transfer is one-dimensional;
(2) thermal, physical, and transport properties are constant; and
(3) fluid phase is incompressible and mass flow rate at every cross-section of packed bed is constant.

Under the assumptions the set of governing equations presented in refs. [7–9] can be reduced to two energy equations for fluid and solid phases:

$$\Pi \rho_f c_f \frac{\partial T_f}{\partial t} + \rho_f c_f v \frac{\partial T_f}{\partial x} = \lambda_{\text{feff}} \frac{\partial^2 T_f}{\partial x^2} + h_{sf} a_{sf} [T_s - T_f], \quad (1)$$

$$(1-\Pi)\rho_s c_s \frac{\partial T_s}{\partial t} = \lambda_{\text{seff}} \frac{\partial^2 T_s}{\partial x^2} - h_{sf} a_{sf} [T_s - T_f], \quad (2)$$

where for the sake of simplicity we write $T_f = \langle T_f \rangle^f$, $T_s = \langle T_s \rangle^s$, $\rho_f = \langle \rho_f \rangle^f$, $\rho_s = \langle \rho_s \rangle^s$, $c_f = (c_p)_f$, $c_s = (c_p)_s$,

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NOMENCLATURE

a_{sf}	specific surface area common to solid and fluid phases [$m^2\ m^{-3}$]
c_p	specific heat at constant pressure [$J\ kg^{-1}\ K^{-1}$]
d	particle diameter [m]
h_{sf}	fluid-to-particle heat transfer coefficient between solid and fluid phases [$W\ m^{-2}\ K^{-1}$]
Nu_{fs}	fluid/solid Nusselt number
t	time [s]
T	temperature [K]
v	velocity of the fluid phase [$m\ s^{-1}$]
w_1, w_2	constants
x	coordinate [m]
$\hat{x}$	coordinate of the position of maximum temperature difference between the solid and fluid phases [m].

Greek symbols

β	constant
ε	dimensionless small parameter
θ	dimensionless temperature, $(T-T_0)/(T_b-T_0)$
λ	thermal conductivity [$W\ m^{-1}\ K^{-1}$]
ξ	dimensionless coordinate
$\hat{\xi}$	dimensionless coordinate of the position of maximum temperature difference between the solid and fluid phases
Π	porosity
ρ	density [$kg\ m^{-3}$]
τ	dimensionless time.

Subscripts

b	boundary
eff	effective property
f	fluid (gas or liquid)
feff	effective property for fluid
max	maximum
0	initial
s	solid
seff	effective property for solid.

Other symbol

$\langle\ \rangle$	local volume average of a quantity.

$v = \langle v_f \rangle$. Here $\langle\rangle$ means local volume average of a quantity and $\langle\rangle^f$ or $\langle\rangle^s$ means intrinsic phase average for fluid or solid phase, respectively [10]. The specific surface area of the packed bed for the fluid phase according to ref. [11] is:

$$a_{sf} = \frac{6(1-\Pi)}{d}. \tag{3}$$

The fluid-to-particle heat transfer coefficient can be estimated according to correlations, established in ref. [12]:

$$\frac{1}{h_{sf}} = \frac{d}{Nu_{fs}\lambda_f} + \frac{d}{\beta\lambda_s}, \tag{4}$$

where $\beta = 10$ for packed bed particles of spherical form.

For a fine structure of a porous bed average particle diameter d is small, so according to equations (3) and (4) $h_{sf}a_{sf}$ takes large values. So for packed beds with small average particle diameter we can introduce an additional assumption:

(4) coefficient $h_{sf}a_{sf}$ in the terms of equations (1) and (2), describing fluid-to-solid heat transfer, is a large parameter.

To apply perturbation technique to the set of equations (1) and (2) we bring them to a dimensionless form. We introduce dimensionless variables:

Temperature $$\Theta = \frac{T-T_s(x,0)}{T_f(0,t)-T_s(x,0)},$$

Distance $$\xi = \frac{\rho_f c_f v}{\lambda_{feff}+\lambda_{seff}} x,$$

Time $$\tau = \frac{(\rho_f c_f v)^2}{(\rho c)_{eff}(\lambda_{feff}+\lambda_{seff})} t,$$

where $(\rho c)_{eff} = \Pi\rho_f c_f + (1-\Pi)\rho_s c_s$.

We assume that the temperature of the solid phase can be represented as:

$$\Theta_s = \Theta_f + \varepsilon\Theta_s^*, \tag{5}$$

where Θ_s^* is a function of coordinate and time, and:

$$\varepsilon = \frac{1}{h_{sf}a_{sf}} \frac{\Pi(\rho_f c_f)^3 (v)^2}{(\rho c)_{eff}(\lambda_{feff}+\lambda_{seff})},$$

is, according to assumption (4), a dimensionless small parameter.

Now the set of equations (1) and (2) can be written as:

$$\frac{\partial\Theta_f}{\partial\tau} + \frac{\partial\Theta_f}{\partial\xi} = \frac{\partial^2\Theta_f}{\partial\xi^2} + O(\varepsilon), \tag{6}$$

$$\Theta_s^* = \frac{\partial\Theta_f}{\partial\tau} + w_1\frac{\partial\Theta_f}{\partial\xi} - w_2\frac{\partial^2\Theta_f}{\partial\xi^2}, \tag{7}$$

where

$$w_1 = \frac{(\rho c)_{eff}}{\Pi\rho_f c_f} \quad w_2 = \frac{\lambda_f(\rho c)_{eff}}{(\lambda_{feff}+\lambda_{seff})\Pi\rho_f c_f}.$$

Equation (6) was obtained by combining equations (1) and (2), and equation (7) is equation (1) in the dimensionless form with regard to equation (5).

SOLUTION OF THE PROBLEM AND INVESTIGATION OF THE TEMPERATURE DIFFERENCE WAVE

Consider a semi-infinite porous bed initially at a uniform temperature, which is suddenly subjected to a step of fluid inlet temperature. Initial and boundary conditions for the function T_f are:

$$T_f(x,0) = T_0 \quad T_f(0,t) = T_b \quad \frac{\partial T_f}{\partial x}(\infty,t) = 0.$$

In dimensionless variables the conditions are:

$$\Theta_f(\xi,0) = 0 \quad \Theta_f(0,\tau) = 1 \quad \frac{\partial\Theta_f}{\partial\xi}(\infty,\tau) = 0. \tag{8}$$

The solution of equation (6) with the initial and boundary conditions (8) can be obtained using Laplace transform methods as:

$$\Theta_f = \frac{1}{2} erfc\left\{\frac{\xi-\tau}{2\sqrt{\tau}}\right\} + \frac{1}{2}\exp\xi \cdot erfc\left\{\frac{\xi+\tau}{2\sqrt{\tau}}\right\}. \quad (9)$$

For high values of τ solution (9) reduces to:

$$\Theta_f = \frac{1}{2} erfc\left\{\frac{\xi-\tau}{2\sqrt{\tau}}\right\}. \quad (10)$$

Solution (10) coincides with the long-time solution obtained in ref. [6] for the single-phase model (no temperature difference between the fluid and solid phases). This solution is in the form of a shock wave propagating from the inlet boundary.

According to equation (7) the function Θ_s^* is:

$$\Theta_s^* = \frac{1-w_2}{4\tau\sqrt{\pi\tau}}\left\{(\xi+\tau)\exp\left[-\left(\frac{\xi-\tau}{2\sqrt{\tau}}\right)^2\right] + (\xi-\tau)\exp\left[\xi-\left(\frac{\xi+\tau}{2\sqrt{\tau}}\right)^2\right]\right\} + (w_1-w_2) \times \left\{\frac{1}{2}\exp\xi\cdot erfc\left[\frac{\xi+\tau}{2\sqrt{\tau}}\right] - \frac{1}{2\sqrt{\pi\tau}}\left[\exp\left\{-\left(\frac{\xi-\tau}{2\sqrt{\tau}}\right)^2\right\} + \exp\left\{\xi-\left(\frac{\xi+\tau}{2\sqrt{\tau}}\right)^2\right\}\right]\right\}. \quad (11)$$

The function Θ_s^* has a singularity at the point $(\xi, \tau) = (0, 0)$ caused by the thermal shock at the boundary $\xi = 0$ at the time $\tau = 0$. Therefore it can be applied to describe the temperature difference between the fluid and solid phases only outside the neighborhood of the point $\tau = 0$.

For high values of τ solution (11) reduces to:

$$\Theta_s^* = \frac{\xi(1-w_2)+\tau(1-2w_1+w_2)}{4\tau\sqrt{\pi\tau}}\exp\left[-\left(\frac{\xi-\tau}{2\sqrt{\tau}}\right)^2\right]. \quad (12)$$

Solution (12) is in the form of a wave localized in space with amplitude decreasing while the wave propagates.

Figure 1 depicts space–time distribution of the function $-\Theta_s^*$. A maximum of the function $-\Theta_s^*$ corresponds to the maximum temperature difference between the fluid and solid phases.

Time dependence of the coordinate $\hat{\xi}$ of this maximum can be found analytically from the equation $(\Theta_s^*)'_\xi = 0$. It is easy to show that for high values of τ this equation is satisfied by the function $\hat{\xi} = \tau$. Substitution of this dependence into equation (12) leads to:

$$(\Theta_s^*)_{\max} = \frac{1-w_1}{2\sqrt{\pi\tau}}.$$

In the dimensional variables that means that for high values of t:

$$\hat{x} = \frac{\rho_f c_f}{(\rho c)_{\text{eff}}} vt,$$

and the maximum temperature difference between the fluid and solid phases is:

$$(T_f - T_s)_{\max} = \frac{T_b - T_0}{2\sqrt{\pi t}}\frac{v}{h_{sf}a_{sf}}\frac{(1-\Pi)\rho_f c_f \rho_s c_s}{(\rho c)_{\text{eff}}^{1/2}(\lambda_{\text{feff}}+\lambda_{\text{seff}})^{1/2}}. \quad (13)$$

From formula (13) it follows that the main factors which influence $(T_f - T_s)_{\max}$ are h_{sf}, a_{sf} and v.

Both of the waves described by equations (10) and (12) propagate with the rate:

$$\hat{v} = \frac{\rho_f c_f}{\Pi\rho_f c_f + (1-\Pi)\rho_s c_s} v,$$

which for the case $\rho_f c_f \neq \rho_s c_s$ does not coincide with the rate of incoming fluid v.

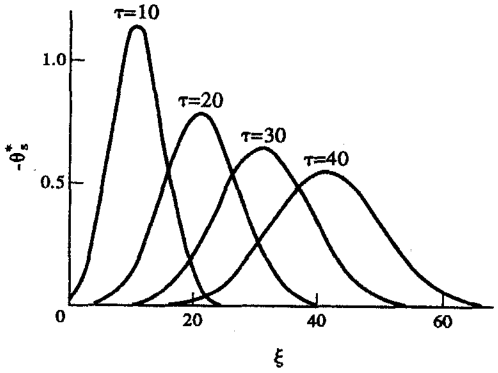

Fig. 1. Calculated wave of temperature difference between the solid and fluid phases as a function of time for $c_f\rho_f = 0.25 c_s\rho_s$, $\lambda_{\text{feff}} = 0.25\lambda_{\text{seff}}$, $\Pi = 0.25$.

CONCLUSIONS

(1) The process of heating of a semi-infinite porous bed by a flow of high-temperature fluid introduces two qualitatively different thermal waves: (a) the temperature of the fluid or the solid phases forms a shock wave; and (b) the temperature difference between the fluid and solid phases forms a thermal wave localized in space.

(2) The waves propagate from the inlet boundary with a rate which for the case of different heat capacities of fluid and solid does not coincide with the rate of the incoming fluid.

(3) The perturbations of the temperature difference between the solid and fluid phases tend to zero at infinity.

Acknowledgement—This research was done while the author was a Research Fellow of the AvHumboldt Foundation at Ruhr-University Bochum.

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